Chapter 1: Concepts of Celestial Navigation

Celestial Navigation is based on principles derived from Coastal Navigation.

1.1 Traditional Coastal Navigation

The method used depends on whether we can take a sight off several land marks at a given moment (fig. 1.1), or whether we are restrained to a single landmark and must use it to plot **running fixes** (fig. 1.2).

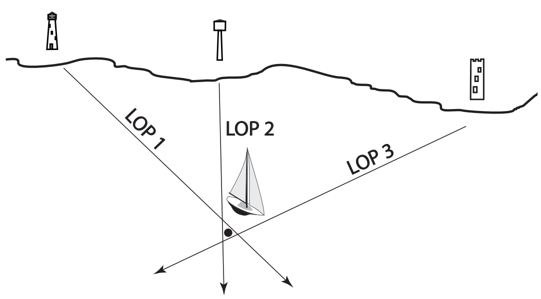


Fig. 1.1 Traditional fix, using three bearings on three landmarks.

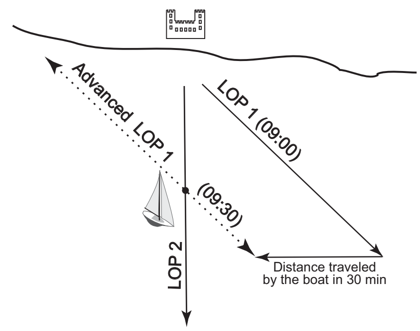


Fig 1.2 Running Fix (Advanced Line of Position), using two bearings on a single landmark.

1.2 Celestial Navigation

The same techniques apply in Celestial Navigation; in this case, the Lines Of Position, or LOPs, are circles of usually very large diameter centered on the **Geographic Position** of the sun (see Section 1.3). Sometimes, we can take concurrent sights on several stars and/or planets and obtain our position at the intersection of two or three LOPs determined by the angle at which we see the celestial bodies above the horizon (fig. 1.3). At other times, we are restricted to the use of a single body, for instance the sun or the moon, in which case we need to take several sights at different times of the day (fig. 1.4). If the boat has moved between sights of a single celestial body, we must **advance** the first LOP by the direction and distance traveled by the boat between the sights in order to obtain a **running fix** (fig. 1.5).

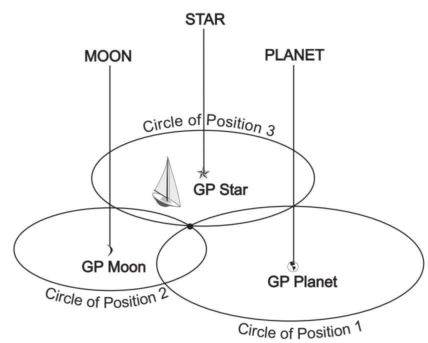


Fig 1.3 A fixed observer can directly use two or three lines (circles) of position from different celestial objects at the same time.

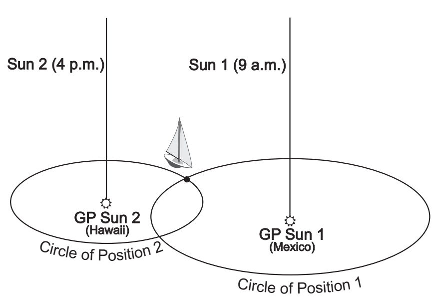


Fig 1.4 A fixed observer can directly use two lines (circles) of position from the same object at different times.

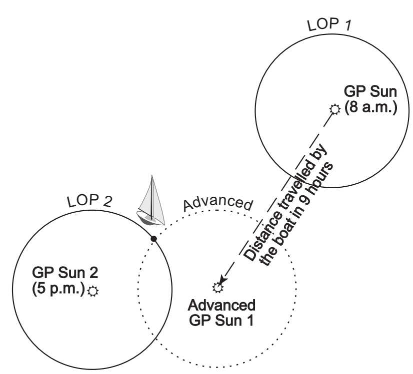


Fig. 1.5 A moving observer needs to **advance** the first line (circle) of position by the distance traveled between the two observations, thus obtaining a Running Fix.

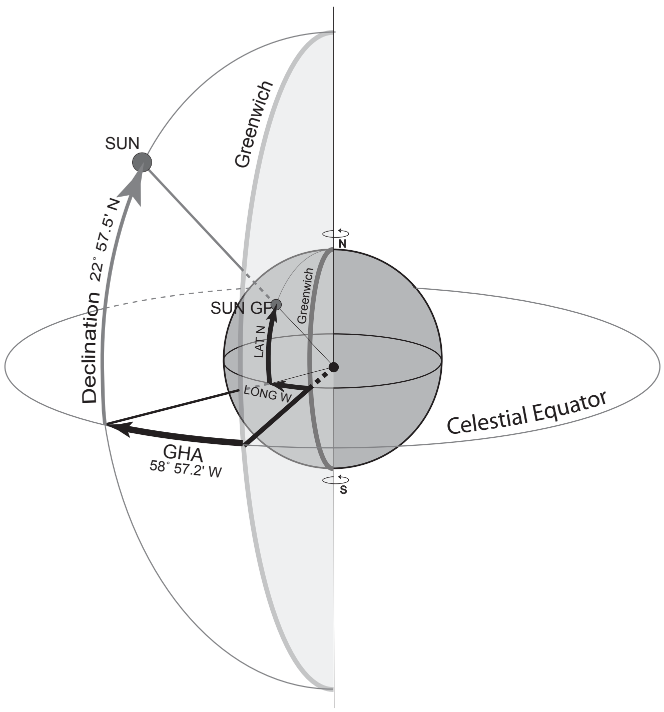
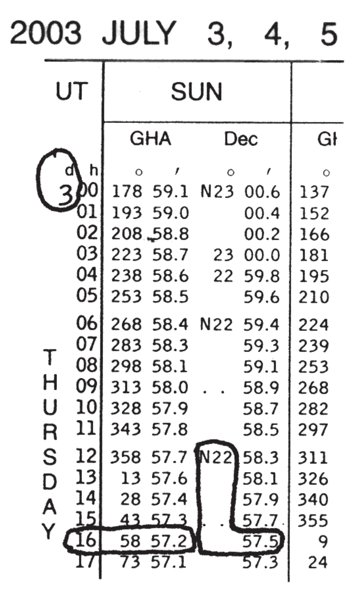


Fig. 1.6 These celestial coordinates are equivalent to the ones on the surface of the earth (Lat. 22° 57.5' N, Long. 58°57.2' W).



2003 JULY 3, 4, 5

	UT	SUN		GH
		GHA	Dec	
	d h	° ′	° ′	°
	3 00	178 59.1	N23 00.6	137
	01	193 59.0	00.4	152
	02	208 58.8	00.2	166
	03	223 58.7	23 00.0	181
	04	238 58.6	22 59.8	195
	05	253 58.5	59.6	210
	06	268 58.4	N22 59.4	224
	07	283 58.3	59.3	239
T	08	298 58.1	59.1	253
H	09	313 58.0	58.9	268
U	10	328 57.9	58.7	282
R	11	343 57.8	58.5	297
S	12	358 57.7	N22 58.3	311
D	13	13 57.6	58.1	326
A	14	28 57.4	57.9	340
Y	15	43 57.3	57.7	355
	16	58 57.2	57.5	9
	17	73 57.1	57.3	24

Fig. 1.7 The Almanac provides the celestial coordinates of the main Celestial Objects. For instance, on July 3, at 16:00 UTC time, Dec for the sun = 22° 57.5' N, and GHA = 058° 57.2'.

1.3 Determining the Geographic Position of a Celestial Body

The Geographic Position (**GP**) of a celestial body at each instant is the point on earth over which it is at that moment (fig. 1.3).

Publications called *Nautical Almanacs* give, for each hour of each day of the year, the exact coordinates of the GP of the sun, moon, major planets, and brightest stars on the celestial sphere. A straight interpolation allows the calculation of the coordinates of the main celestial bodies at any time in between (fig. 1.6).

In the Almanac, the coordinates of celestial objects are given in terms of **Greenwich Hour Angle** (or **GHA**), the equivalent of a longitude; and **Declination** (or **Dec**), the equivalent of a latitude. The coordinates of the celestial bodies on the celestial sphere, in degrees of Greenwich Hour Angle and Declination, are the same as the coordinates of their GP on the surface of the earth. The only difference is that GHA is counted, very logically, from the Greenwich Meridian towards the west, all the way around the earth up to 360°; a longitude, by contrast, is either west or east of the Greenwich Meridian, to a maximum of 180° (fig. 1.6).

For instance, the *2003 Almanac* tells us that, on the 3rd of July of that year, at precisely 16:00 UTC (Greenwich time), the sun Dec was 22° 57.5' N, and its GHA was 058° 57.2' (fig. 1.7; full table in Appendix 2, Almanac daily table for July 3, 4 and 5, p. A2-9). This means that, at this precise moment, the sun was exactly above a point on earth located at 22° 57.5' of Latitude North, and 058° 57.2' of Longitude West. This is in the West Atlantic near the Tropic of Cancer, some 1,000 km NE of Puerto Rico. In other words, if our boat had been at that spot, i.e. at the sun's GP, we would have seen the sun exactly overhead, at our **zenith**, i.e. at an **altitude** of 90° above the horizon.

The next day, in the morning of July 4 at 02:00 UTC, the sun's GHA was 208° 56.1' (Appendix 2, p. A2-9), and returning towards Greenwich from the east. The longitude of its GP was then 360° 00.0' – 208° 56.1' = 151° 03.9' E.

The same type of information is available for each of the celestial objects listed in the Almanac: the sun, the moon, the four main planets, and the 57 stars used for navigation. If our boat had been, instead, in the Pacific Ocean off the shores of Vancouver Island, we would have seen the sun at a lower angle H above the horizon.

1.4 Plotting the Circles of Position around the GP

Since the earth is approximately spherical, we can tell how far we are from the **Geographical Position** or **GP** of a celestial body, i.e. from the point on earth where the body appears to be directly overhead at the time of the sight. The **altitude** H (for **Hauteur**: the notations are of French origin) of the celestial object over the horizon determines how far away we are from its GP. The further away our boat is from the sun GP at that particular time, the smaller its altitude, i.e. the angle of the sun over the horizon (fig. 1.8).

If we measure this angle H with a sextant, it is relatively simple to calculate how far away we are from the GP of the sun: this distance is the radius of our Circle of Position. On charts large enough to cover most of the Pacific Ocean, and to include both our boat position and the sun's GP, we could draw a circle around the sun's GP with a radius corresponding to our calculated distance from the GP. This would give us a first **circle of position**: our boat would be somewhere along the circle, centered on the GP, from which the sun could be seen at this angle H above the horizon. Subsequent sights of the sun would give us our position at the intersection of two or three circles of position.

The drawback of this method is that it is not very precise: a chart on which we could plot both the boat and the GP of the sun would necessarily be of such small scale, covering huge areas of the earth, that our position would be quite approximate. If we were sailing back from Hawaii to Victoria, for instance, the intersection of two or three circles of position would probably not be precise enough to tell us whether we were approaching the southern part of Vancouver Island or the Northern part of the State of Washington.

In this manual, we will treat the Almanac coordinates of the celestial bodies, and in particular of the sun, as if they were the geographical coordinates of those bodies' GPs on the surface of the earth. This greatly simplifies the comprehension of the method. We simply replace the centers of the stars and planets by small lights on the surface of the earth, just ***underneath*** *each celestial body. For an observer at the center of the earth, from which all measurements are made, the picture of the sky is the same.*

We also treat the apparent movements of the celestial bodies across the celestial sphere as if they were real. In other words, we use the Ptolemy model of the universe, and assume that the earth is at its center. Since all movements are relative, the model that we use to represent the various orbits does not change the calculations or the results, but the Ptolemy model makes it considerably simpler to imagine the movements. For instance, we will be referring to the ***revolution of the sun around the earth****, when this movement is only apparent.*

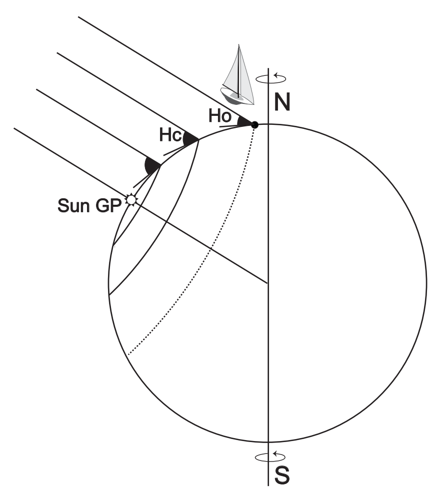


Fig. 1.8 Circles of Position for various **altitudes** H of a celestial object over the horizon. The further away the observer is from the GP, the lower the object's altitude over the horizon.

1.5 The Marcq Saint Hilaire Solution

In 1874, an astute French naval commander, Adolph Laurent Anatole Marcq de Blond de Saint Hilaire, thought of a way to avoid the problem of drawing circles of position centered on the sun's GP several thousand miles away. Instead of trying to draw the whole circle of position, or at least the relevant sector of this circle, he had the idea of assuming a position for the boat from **Ded Reckoning** (**ded** meaning **deduced** position, using calculations from heading, speed and time), and calculating at what angle above the horizon, and in what direction, the sun or other celestial bodies would be when seen from this assumed position.

A navigator would then be able to compare this **calculated** altitude of the sun, Hc, with Ho, the altitude **observed** with the sextant; this would tell him or her how far off the boat was from its assumed position, either towards the sun or away from it. In other words, Marcq Saint Hilaire used a differential method, measuring and plotting small differences between calculated and measured angles, rather than an absolute method, trying to draw a sector of the huge circle of position centered on the GP of the sun (or any other celestial body).

1.6 Sight Reduction Tables

The Marcq Saint Hilaire method requires the use of Sight Reduction tables in order to calculate the altitude of the sun at the assumed position. The simplest ones to use are those published by the National Imagery and Mapping Agency (U.S.), newly referred to as *Pub. No. 249* in North America, and *AP 3270* in the UK. They were originally prepared for air navigation in the early sixties by the Hydrographic Office, which explains that they are still often known as *H.O. 249*.

Once the coordinates of the GP of a celestial object are calculated from the Almanac, by interpolation to the nearest second for the precise time of the sight, the *Sight Reduction Tables* in *Pub. No. 249* allow the calculation of the angle of that object above the horizon, and its **bearing** (direction) as seen from any assumed position on earth. The *Sight Reduction Tables* cover a sector of the celestial sphere extending from 30° N to 30° S of the equator, which includes the sun, the moon, the planets, and some of the main stars.

Other Sight Reduction tables, such as *H.O. 229*, are specifically designed for marine navigators; they cover the whole celestial sphere but are considerably bulkier. Together with the concise *Sight Reduction Tables* at the end of the *Nautical Almanac*, however, they are traditionally perceived as more difficult to use than those in *Pub. No. 249*.

1.7 Angle at the Center of the Earth

There is a direct relationship between the altitude of a celestial object above the horizon (H) and the corresponding angle at the center of the earth (90° – H, referred to as the **Zenith Distance ZD**; see fig. 1.9). This angle at the center of the earth determines the distance, on the surface of the earth, between the Geographical Position of the celestial body, GP, and the boat. Our **Line of Position** is this circle (fig. 1.10). The original definition of the Nautical Mile was that one minute of angle at the center of the earth marked an arc of one nautical mile on the surface. The new definition relates the nautical mile to the metric system: 1 NM = 1,852 m.

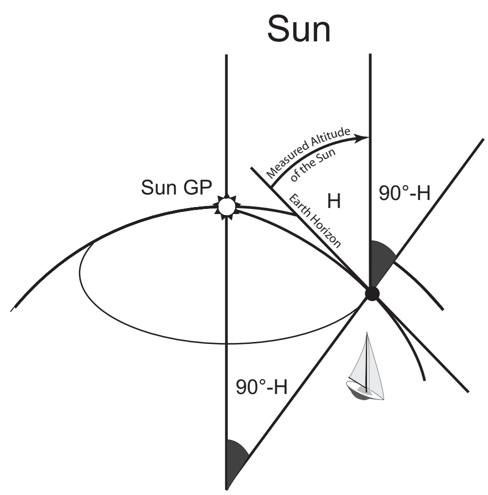


Fig. 1.9 The angle at the Center of the Earth (90° – H) is equal to the Zenith Distance, i.e. 90° minus the altitude (H) of the sun above the horizon.

1.7.1 Example

Captain Cook took a sight on the sun, on June 21, 1769. From his *Nautical Almanac*, he knew that, at the time of the sight, the sun was over a point just east of Mazatlan, at Lat. 23° N, Long. 105° W. With his sextant, he measured the altitude of the sun as Ho = 30° above the horizon. What was the radius of the Circle of Position, centered on the sun's Geographic Position, on which his boat was located?

Answer: The Zenith Distance is 90° – 30° = 60°.

His Circle of Position, centered on the sun's GP in Mexico, has a radius of 60° x (60' per °) = 3,600 NM.

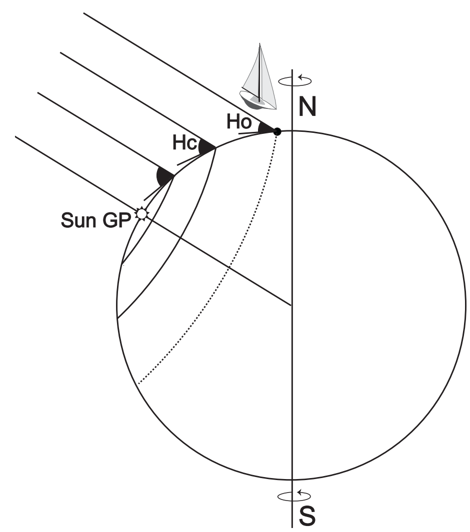


Fig. 1.10 The angle at the Center of the Earth (90° – H) allows drawing a Circle of Position, at least in theory.

1.8 Calculation of the length of a segment of Great Circle

The distance between two points on the surface of the earth can be calculated directly without a sextant. While there are several formulae to determine the length of an arc of Great Circle between two points of coordinates Lat1, Long1 and Lat2, Long2, the most traditional one is:

$$D \text{ (in NM)} = \text{ArcCos } [(\text{Sin Lat1} \times \text{Sin Lat2}) + (\text{Cos Lat1} \times \text{Cos Lat2} \times \text{Cos } (\text{Long2} - \text{Long1}))] \times 60$$

In this formula, the angles of Lat. and Long. are in radians. Degrees and minutes of angle can be converted into radians from the relation 1° = (π / 180) radians = 3.1416 / 180 = 0.017 radian. Latitudes south and longitudes west are identified with a **minus** sign. The notation **ArcCos** is often represented as **Cos^{-1}**

In our example, we know from his diary that Captain Cook was near Tahiti, at Lat. 18° S, Long. 150° W. The calculations for the distance between his boat and the sun's GP look like this:

Lat1 = 23° N = 0.401426 radians; Sin Lat1 = Sin 0.401426 = **0.390731**; Cos Lat1 = **0.920505**

Lat2 = 18° S = –0.314159 radians; Sin Lat2 = Sin – 0.314159 = **–0.309017**; Cos Lat2 = **0.951057**

Long2 – Long1 = 150° – 105° = 45° = 0.785398 radians; Cos (Long2 – Long1) = **0.707107**

$$
\begin{aligned}
D \text{ (in NM)} &= \text{ArcCos}\,[(0.390731 \times (-0.309017)) + (0.920505 \times 0.951057 \times 0.707107)] \times 60 \\
&= \text{ArcCos}\,[-0.120743 + 0.619039] \times 60 \\
&= \text{ArcCos}\,[0.498296] \times 60 \\
&= 1.049164 \text{ radian} \times 60 \\
&= 60° \times 60 = 3{,}600 \text{ NM}
\end{aligned}
$$

1.9 Navigation by latitude and the problem of longitudes

Long before Almanacs and Sight Reduction tables were produced, and even before the sextant had been invented, mariners used to sail across seas and oceans by following a constant parallel of latitude. They could do this simply by checking the sun at noon every day, when it is highest over the horizon, or the polar star at dawn or dusk. Christopher Columbus and Magellan, for instance, used this method to navigate. The method is still used today (see Chapter 6, *Latitude by Noon Sight*, and Chapter 8, *Approximate Latitude from Polaris*).

The determination of one's longitude around the earth, however, had always eluded navigators until fairly recently. One naturally tries to correlate the longitude of the boat with the apparent movement of the sun or other celestial bodies around the earth, and this requires a very accurate measure of time.

Since the sun appears to turn around the earth in a day (360° in 24 hours, or 15° per hour), we could tell our position away from a reference meridian by noting the exact moment at which the sun crosses the meridian of the boat: the time which the sun takes to travel from the reference meridian to the meridian of the boat, at 15° per hour westward, determines our longitude. The meridian of Greenwich was selected as the international reference meridian in 1884.

For example, if the sun is highest over the meridian of our boat at 13:00 UTC (1:00 p.m. Greenwich time) on a day when it crossed the meridian of Greenwich at 12:00, we are **one hour** or 15° west of this reference meridian. If the sun is highest over our boat at 4:20 p.m., we are (4 + 1/3 h) x 15°/h = 65° west of the meridian of Greenwich.

Navigators have been able to keep a precise track of time since the invention of the chronometer at the end of the eighteenth century. Chapter 3 explains how time is determined and measured, thus enabling us to determine our longitude.