Chapter 2: Measure of the Sun’s Altitude: Ho

The altitude (angle) of the sun above the horizon is traditionally measured with a sextant.

2.1 Principle of the Sextant

The sextant measures the angle between what is seen straight through the clear window of the sextant, on the left side of the eyepiece, and what is seen by reflection on two mirrors, on the right side of the eyepiece. In the case of celestial navigation, the observer measures the angle between the horizon, seen straight through the sextant, and a celestial body, seen in the two mirrors. Sights on a celestial body are taken either on its center if it is punctual such as a star or a distant planet or, traditionally, on the bottom point of its Lower Limb if it has an apparent diameter, such as the sun or the moon. This is shown in fig. 2.1. Other uses of the sextant are described in Chapter 19.

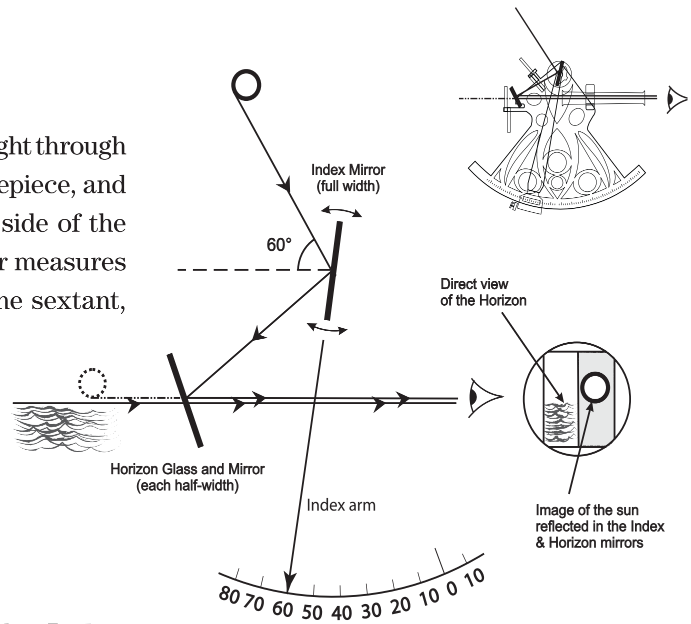


Fig. 2.1 To take a sextant sight, the observer turns the index mirror until the reflected image of the celestial body can be seen exactly on the horizon.

The degrees of angle are measured directly off the **Index** (fig. 2.2); the minutes of angle off the **drum**; and the 1/20th of minutes (or 1/10th of minutes on some sextants) off the **Vernier**. The Vernier provides an expanded scale, easier to read. The angle thus measured, before any correction, is typically called **Hs** (Hauteur Sextant).

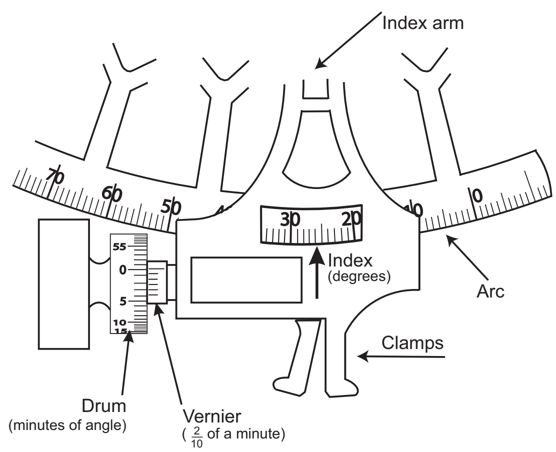


Fig. 2.2 The degrees of **altitude** of the celestial object above the horizon are read on the arc of the sextant, behind the **index**; the minutes are read on the **drum**, and the tenth of minutes on the **Vernier**.

On the Vernier, the graduations for the decimals of minutes of angle, 0.1' or 0.2', are 20% shorter (10% on some sextants) than the graduations for the minutes of angle on the drum: five (or 10) graduations on the Vernier are the length of four (or nine) graduations on the drum (fig. 2.3 and 2.4). The decimals are measured where graduations coincide on the Vernier and the drum.

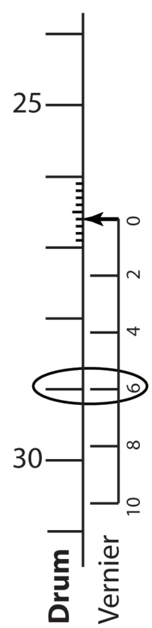


Fig. 2.3
XXX° 26.6'
(exact measure)

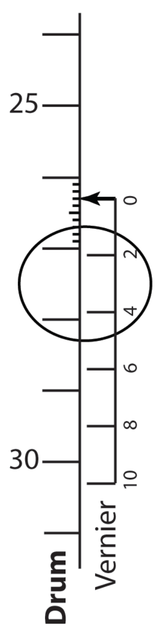


Fig. 2.4
XXX° 26.3'
(estimated, between 2 and 4)

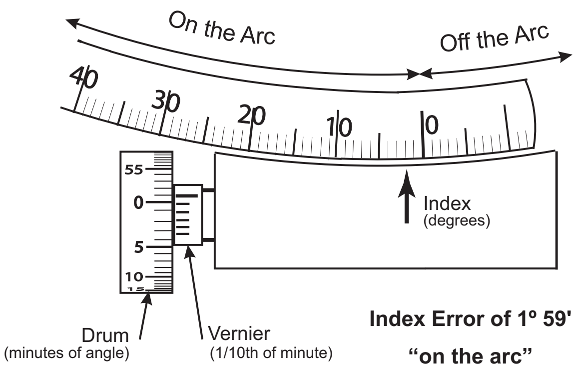


Fig. 2.5 When the observer takes a sight of the horizon, the altitude should read 0. Any difference, or **index error**, needs to be noted and taken into account.

2.2 Index Error

Some errors on the sextant are inherent to its construction. These include warping, slackness of the center bearing, concave or convex mirrors, or poor alignment of the eye-piece. A number of other errors, such as the perpendicularity error and the side error, can usually be corrected by adjusting the mirrors.

One error that can be measured but cannot always be fully corrected is the Index Error, caused by the offset of the zero of the graduation, usually a few minutes of angle. This is evident when looking at the horizon and bringing side by side the horizon seen directly though the telescope with the horizon seen after reflection on the index and horizon mirrors. The Index Error can be read directly **on the arc** of the sextant, or **off the arc** along its extension.

The Index Error needs to be measured regularly and taken into account at every sight. If it is **on the arc**, the zero of the arc of the sextant is too far forward (away from the observer). The illustration (fig. 2.5) shows an Index Error of 1° 59' on the arc: any angle read on the sextant will be too large by 1° 59'. The true angle of the sun above the horizon will be obtained by subtracting the index error from the angle read at the index.

This is easy to remember: when we step on a weight scale that already shows 5 kg, i.e. if it has an error of 5 kg **on the scale**, we need to remove this error from any weight read on the scale. Conversely, if the error is 3 kg **off the scale**, i.e. if it takes a weight of 3 kg just to get the scale to read zero, then we will need to add this error to any measurement. On a sextant, if the Index Error is **off the arc**, we need to add it; if it's **on the arc**, we need to subtract it. Some navigators use the mnemonic formula: **if it's on, take it off; if it's off, put it on.**

If the Index Error is	**it is correct by:**
On the arc	Subtracting the error
Off the arc	Adding the error

2.2.1 Example 1: Index error on the arc

Measured sextant altitude Hs	45° 22.0'
Index error: 4' on the arc	– 4.0'
Corrected sextant altitude Hs	45° 18.0'

2.2.2 Example 2: Index error off the arc

Measured sextant altitude Hs	39° 58.0'
Index error: 6' off the arc	+ 6.0'
Corrected sextant altitude Hs	40° 04.0'

A number of relatively small errors need to be corrected. The sextant or **Index** error is mechanical and due to minor imperfections in the construction of the sextant. The **Dip** error is caused by the height of the eye above the horizontal plane, which causes the observed angle between the horizon and the celestial body to be slightly too large. Finally, the **Main** error combines several errors: the **Half Diameter** error, caused by the fact that the sun's altitude over the horizon is traditionally measured to the bottom of the Lower Limb instead of its center; the **Parallax** error, due to the location of the observer off to the side of the earth rather than at the Geographical Position of the observed celestial object; and the atmospheric or **Refraction** error, which tends to be larger when the celestial objects are lower above the horizon.

Summary of Dip Angles for various heights of observation above sea level

Height of the eye (m)	(ft)	**Dip angle**
0	0	00.0'
5	16	–03.9'
10	33	–05.6'
20	66	–07.9'
30	97	–09.6'
40	130	–11.1'
50	162	–12.4'

Fig. 2.6

2.3 Dip

The altitude of the sun which we measure above the observer's horizon is always too high, and this Dip error always needs to be subtracted, hence the "**–**" sign in front of it (fig. 2.6). This is because the observer is standing on the deck of the boat, and thus well above the surface of the ocean when taking a sextant sight. As a result, our **observer's horizon** is further away, and lower than the **rational** horizon which we would use if our eye was just above water, for instance if we were looking at the horizon from the periscope of a submarine (fig. 2.7). The higher the observer is above the surface of the ocean, the larger the Dip angle (fig. 2.8).

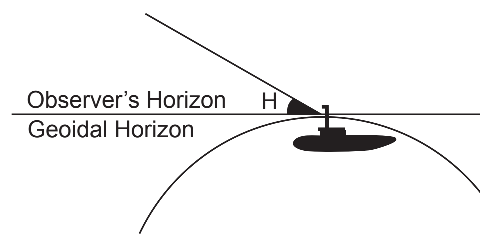


Fig. 2.7 The *Nautical Almanac* and the *Sight Reduction Tables* assume that the sights are taken at sea level, for instance from the periscope of a submarine barely emerging above the water.

A detailed Dip-angle table is given in the front page of the *Nautical Almanac* (extract in fig. 2.9; full table in Appendix 2, p.1). It gives the Dip in minutes of angle (') for various heights of the observer's eye above sea level. The right-hand side of the table is divided into two parts: in the upper part, the corrections are given for heights in metres, first for heights from 1.0 m to 3.0 m and, further down, for heights from 20 m to 48 m, in rather large increments. In the lower part, the corrections are given for heights expressed in feet.

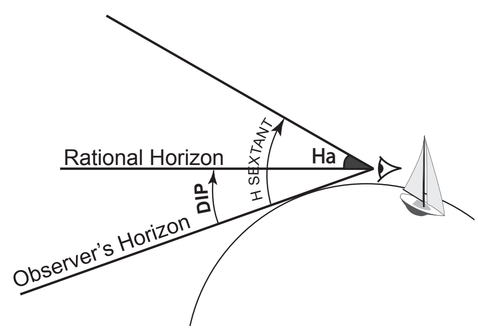


Fig. 2.8 The height of the observer's eye above sea level determines the **Dip**. This angle must be removed from the altitude of the celestial body above the horizon that is measured with a sextant (H sextant) to obtain the Apparent Altitude (Ha).

The most common heights for an observer's eye above sea level are between 2.4 and 21.4 m (8.0 and 70.5 ft) above sea level. The corresponding Dip angles are given on the left side of the table, in small increments.

Note that for a height of exactly 3 m, for instance, the left-hand side of the table leaves the ambiguity of having to choose between a correction of –3.0' and –3.1'. The right-hand side of the table clarifies that, for 3.0 m, the correction is –3.0'. This indicates that, for a height exactly at the end of each range, for instance 6.3 m, the lower correction (–4.4') should be applied.

DIP

Ht. of Eye	Corr^n	Ht. of Eye
m	'	ft.
2.4		8.0
	−2.8	
2.6		8.6
	−2.9	
2.8		9.2
	−3.0	
3.0		9.8
	−3.1	
3.2		10.5
	−3.2	
3.4		11.2
	−3.3	
3.6		11.9
	−3.4	
3.8		12.6
	−3.5	
4.0		13.3
	−3.6	
4.3		14.1
	−3.7	
4.5		14.9
	−3.8	
4.7		15.7
	−3.9	
5.0		16.5
	−4.0	
5.2		17.4
	−4.1	
5.5		18.3
	−4.2	
5.8		19.1
	−4.3	
6.1		20.1
	−4.4	
6.3		21.0
	−4.5	
6.6		22.0
	−4.6	
6.9		22.9
	−4.7	
7.2		23.9
	−4.8	
7.5		24.9
	−4.9	
7.9		26.0
	−5.0	
8.2		27.1
	−5.1	
8.5		28.1
	−5.2	
8.8		29.2
	−5.3	
9.2		30.4
	−5.4	
9.5		31.5
	−5.5	
9.9		32.7
	−5.6	
10.3		33.9
	−5.7	
10.6		35.1
	−5.8	
11.0		36.3
	−5.9	
11.4		37.6
	−6.0	
11.8		38.9
	−6.1	
12.2		40.1
	−6.2	
12.6		41.5
	−6.3	
13.0		42.8
	−6.4	
13.4		44.2
	−6.5	
13.8		45.5
	−6.6	
14.2		46.9
	−6.7	
14.7		48.4
	−6.8	
15.1		49.8
	−6.9	
15.5		51.3
	−7.0	
16.0		52.8
	−7.1	
16.5		54.3
	−7.2	
16.9		55.8
	−7.3	
17.4		57.4
	−7.4	
17.9		58.9
	−7.5	
18.4		60.5
	−7.6	
18.8		62.1
	−7.7	
19.3		63.8
	−7.8	
19.8		65.4
	−7.9	
20.4		67.1
	−8.0	
20.9		68.8
	−8.1	
21.4		70.5

Ht. of Eye	Corr^n
m	'
1.0	−1.8
1.5	−2.2
2.0	−2.5
2.5	−2.8
3.0	−3.0
See table ←	
m	'
20	−7.9
22	−8.3
24	−8.6
26	−9.0
28	−9.3
30	−9.6
32	−10.0
34	−10.3
36	−10.6
38	−10.8
40	−11.1
42	−11.4
44	−11.7
46	−11.9
48	−12.2
ft.	'
2	−1.4
4	−1.9
6	−2.4
8	−2.7
10	−3.1
See table ←	
ft.	'
70	−8.1
75	−8.4
80	−8.7
85	−8.9
90	−9.2
95	−9.5
100	−9.7
105	−9.9
110	−10.2
115	−10.4
120	−10.6
125	−10.8
130	−11.1
135	−11.3
140	−11.5
145	−11.7
150	−11.9
155	−12.1

Fig. 2.9 Corrections for the height of the eye above sea level (Dip).

2.3.1 Example 1: Height of the eye in metres

Sextant altitude Hs: 32° 54.4'
Height of eye: 2.5 m

From the Almanac: Dip for an eye height between 2.4 m and 2.6 m: –2.8'
Corrected sextant altitude:

Hs	32° 54.4'
Dip	– 2.8'
Ho	32° 51.6'

2.3.2 Example 2: Height of the eye in feet.

Sextant altitude Hs: 18° 39.6'
Height of eye: 10 ft

From the Almanac: Dip for an eye height between 9.8 feet and 10.5 feet: –3.1'
Sextant altitude corrected for height of eye (Dip):

Hs	18° 39.6'
Dip	– 3.1'
Ho	18° 36.5'

The two small corrections discussed above, the Index Error **IE** and the Dip **D**, need to be applied to the angle measured by the observer with the sextant. The angle thus corrected for IE and Dip is traditionally called **Ha** (for Hauteur Apparente).

2.4 Main Correction

The **Main** correction is a group of further corrections which need to be applied before the observer has an accurate measurement of the angle between the celestial body and the horizon. It is given for the sun and for stars and planets (Appendix 2, p. 1). Corrections for the moon are given in separate tables (Appendix 2, p. 29 and 30).

In the case of the sun, the biggest component of the Main Correction compensates for the fact that the sight is on the edge of the body (typically the Lower Limb) rather than on its center. The Main Correction for the sun, for example, is typically between 11' and 16' for a sight on the Lower Limb, depending on how high the sun is above the horizon. Most of this correction represents the semidiameter of the sun (approximately 16').

There are two other components of the Main Correction; they apply to the sun, the stars and the planets. One of them compensates for atmospheric refraction, especially when the celestial object is relatively close to the horizon. The other, very small, compensates for the fact that the sight is taken from a point on earth that is not usually directly under the celestial object: this introduces a small **parallax** error. These three components of the Main Correction are explained below in Sections 2.5, 2.6, and 2.7.

The table called *Altitude Correction Table 10°–90° Sun, Stars, Planets*, in the Almanac (Appendix 2, p. 1), gives the sum of all the components of the main correction.

2.5 Correction from Lower Limb (or Upper Limb) to the Center of the Sun

The diameter of the sun, as seen from earth, varies slightly around 32' depending on the time of year. The Almanac gives the sun's semidiameter for each day of the year, on the bottom-left corner of each daily page, under **SD** (see for instance Appendix 2, p. 4 and following). The semidiameter of the sun varies from a high of 16.3' in December and January, to a low of 15.8' between May and August. Since the sextant measures the angle between the horizon and the Lower Limb of the sun (Ha, the **apparent** angle), rather than its center (Ho, the **observed** angle), we need to add one semidiameter, or approximately 16', to the sextant altitude. Figure 2.10 shows the semidiameter correction for the sun.

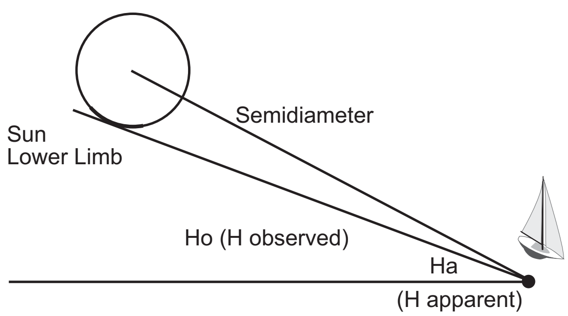


Fig. 2.10 Using the sextant, the observer usually measures the apparent angle Ha between the horizon and the **Lower Limb** of the sun. This needs to be adjusted by half the diameter of the sun in order to obtain the observed altitude Ho of the center of the sun.

This correction for the semidiameter of the sun accounts for most of the **Main Correction** given in the Almanac in two tables, one for the winter months (Oct.–Mar.; maximum total correction: 16.1'), and one for the summer months (Apr.–Sept.; maximum correction: 15.9'). The Main Correction is shown on the left-hand side of the *Altitude Correction Table* (see extract in fig. 2.15; full table in Appendix 2, p. 1).

2.5.1 Example 1

Date of sight:	July 03, 2003
Apparent sextant altitude Ha:	28° 48.9'

Semidiameter **SD** of the sun on July 03, from the Almanac (Appendix 2, p. 9, bottom left corner): 15.8'

Observed altitude Ho, after correction from Lower Limb to center of the sun:

Ha	28° 48.9'
Semidiameter	+ 15.8'
Ho	29° 04.7'

2.5.2 Example 2

Date of sight:	November 20, 2003
Apparent altitude Ha:	17° 28.2'

Semidiameter of the sun on Nov. 20, from the Almanac (Appendix 2, p. 14): 16.2'

Altitude Ho, observed after correction from Lower Limb to center of the sun:

Ha	17° 28.2'
Semidiameter	+ 16.2'
Ho	17° 44.4'

In some cases, for instance when the Lower Limb of the sun is obscured by clouds, we need to take our sight on the Upper Limb. When this occurs, we need to subtract the half diameter of the sun to obtain the **Observed Altitude**.

In the narration of his extraordinary trip through the Antarctic Ocean in a small wooden row boat from Elephant Island to South Georgia, in April 1916, Frank Worsley, the captain and navigator of Shackleton's crushed ***Endurance***, reports that he finally saw the sun after 13 days at sea, but through "mist" and "thick haze". This forced him to take a sextant noon sight using the estimated center of the sun. Reading Ho directly, after minor corrections, he didn't need to correct for the sun's semidiameter; however, the imprecision caused by the non-availability of the Lower and Upper Limbs of the sun obliged Worsley to report to Shackleton that he "could not be sure of (their) position to 10 miles".

Shackleton then decided to maintain a course towards the middle of South Georgia, rather than aim towards the NW point of the island and going around it, which would have entailed the risk of missing the island altogether. He thus aimed towards the south side of the island, where he eventually landed. This landing on the wrong shore forced the cold and exhausted men to cross a range of mountains on foot to one of the whaling stations, which were all established on the more sheltered north shore.

2.6 Correction for Refraction

Refraction from the atmosphere, when the sun is close to the horizon, can introduce significant errors, and sights should be avoided when the sun is less than 10° above the horizon. When the sun is 10° above the horizon, the refraction angle is 5.1' (fig. 2.11). This means that the sun appears to be 5.1' higher above the horizon than it really is, and a sight taken with the sextant needs to be corrected by subtracting 5.1'. When the sun is 20° above the horizon, the refraction error drops to 2.5'. For apparent altitudes of the sun higher than 45°, the refraction angle is smaller, dropping to near zero when the sun is nearly above the observer's zenith (fig. 2.12).

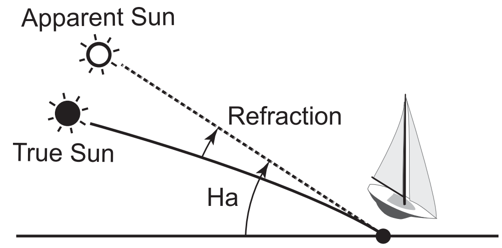


Fig. 2.11 Refraction makes the celestial bodies seem higher above the horizon than they really are, especially when they are close to the horizon.

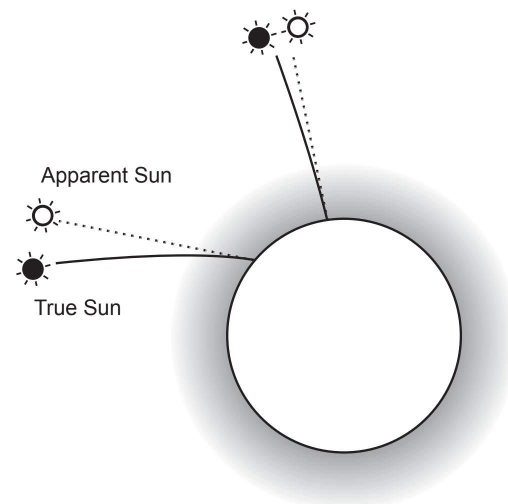


Fig. 2.12 Atmospheric refraction is highest when the celestial object is low above the horizon.

2.6.1 Example 1: Sun low above the horizon

Apparent altitude Ha: 14° 17.8'

Refraction angle for an object 14° above the horizon: 3.8' (from tables not reproduced here)

Observed altitude after correction for refraction:

Ha	14° 17.8'
Refr.	– 3.8'
Ho	14° 14.0'

2.6.2 Example 2: Sun high above the horizon

Apparent altitude Ha: 72° 18.3'

Refraction angle for an object 72° above the horizon: 0.4' (from tables not reproduced here)

Observed altitude after correction for refraction:

Ha	72° 18.3'
Refr.	– 0.4'
Ho	72° 17.9'

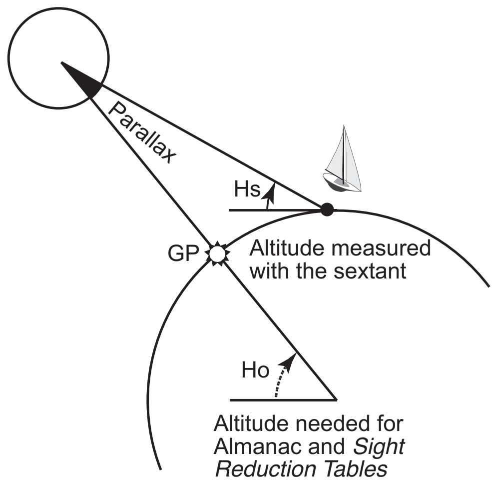


Fig. 2.13 Parallax is the very small angle under which an observe, at the center of a celestial body, would see the boat and the center of the earth.

2.7 Correction for Parallax

The astronomical tables used for celestial navigation represent the stars and planets by the position of their centres. In other words, the celestial coordinates given in the Almanac, and the **sight reduction** procedure, assume that the observer is on the line between the center of the celestial object and the center of the earth. This only happens when the object is exactly overhead, at the observer's zenith. In this case, the observer is at the GP of the celestial body. Most of the time, however, the observer is not directly below the observed celestial object, and the angles of observation need to be **reduced**, i.e. corrected, to the angles between the center of the celestial body and that of the earth.

Parallax is the very small angle, as seen from the center of the celestial body, between the observer on earth and the center of the earth (or the GP of the celestial body; fig. 2.13). It only applies to celestial bodies close to earth: the moon, the sun, and the closest planets.

By adding half a diameter of the sun to our angle of sight, we have already corrected the angle at which we see the sun above the horizon, from the one measured off the Lower Limb to the one at the center of the sun. In order to correct for parallax, we need now to do the same from the other end, for instance from the sun or the moon, and correct for the angle between the observer and the center of the earth, as seen from the center of the sun.

When the sun is close to the observer's zenith (Ha close to 90°), the sight to the sun is a line going through the GP and the center of the earth, and there is little or no parallax correction to make (fig. 2.14, left side).

If the sun is very close to the horizon, on the other hand, the observer would appear, from the sun, to be at the edge of the earth, and the parallax error would be maximal, and close to a semidiameter of the earth (fig. 2.14, right side).

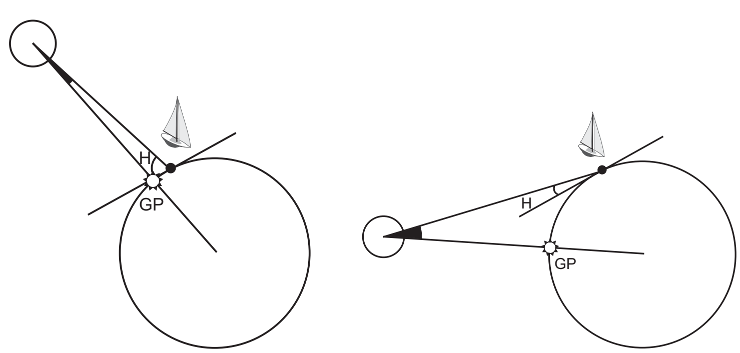


Fig. 2.14 Parallax (black angle) is very small when the celestial object is high on the horizon (left); it increases as the object is closer to the horizon of the observer (right).

For the moon, parallax is particularly important because our satellite is much closer: from the center of the moon, the earth appears large, and an observer on one side or the other of the earth needs to be **reduced** to the earth center or to the GP of the moon on the surface of the earth.

For the planets nearest to Earth (Venus and Mars), we need to make a small correction for parallax, up to 0.5' for Venus depending on the time of year. For objects that are much further away, for instance the stars, this angle of parallax is so small that it can be ignored.

The Almanac gives two detailed *Main Correction* tables for the sun; one for the winter months, and one for the summer. These tables include the corrections for the semidiameter of the sun (*Upper Limb* or *Lower Limb*); refraction as a function of the Apparent Altitude Ha (column *App. Alt.*), and parallax according to the time of year (**Oct.–Mar.**; **Apr.–Sept.**): the orbit of the earth around the sun is an ellipse, and the apparent diameter of the sun therefore changes slightly throughout the year. An extract of this table is given to the right (fig. 2.15) The full table is given in Appendix 2, p. 1.

In the case of a sight on the Lower Limb, and for low altitudes (10° above the horizon) between April and September, we would need to add 16' to correct to the center of the sun, and subtract 5.3' for refraction; the net correction would be +10.7', which is the pre-calculated result given in the right column on the *Main Correction Table*, under *Lower Limb*, *Apr.–Sept.*

If we had taken a sight on the Upper Limb under the same circumstances, we would have needed to subtract 16' to correct our sight from the Upper Limb and another 5.1' to correct for refraction, for a total of –21.1' (see the right column on the *Main Correction Table*, under *Upper Limb*). Note that refraction on the Upper Limb, higher above the horizon, is very slightly less than on the Lower Limb.

OCT.—MAR.	SUN		APR.—SEPT.		
App. Alt.	Lower Limb	Upper Limb	App. Alt.	Lower Limb	Upper Limb
° '	'	'	° '	'	'
9 34			9 39		
	+10·8	−21·5		+10·6	−21·2
9 45			9 51		
	+10·9	−21·4		+10·7	−21·1
9 56			10 03		
	+11·0	−21·3		+10·8	−21·0
10 08			10 15		
	+11·1	−21·2		+10·9	−20·9
10 21			10 27		
	+11·2	−21·1		+11·0	−20·8
10 34			10 40		
	+11·3	−21·0		+11·1	−20·7
10 47			10 54		
	+11·4	−20·9		+11·2	−20·6
11 01			11 08		
	+11·5	−20·8		+11·3	−20·5
11 15			11 23		
	+11·6	−20·7		+11·4	−20·4
11 30			11 38		
	+11·7	−20·6		+11·5	−20·3
11 46			11 54		
	+11·8	−20·5		+11·6	−20·2
12 02			12 10		
	+11·9	−20·4		+11·7	−20·1
12 19			12 28		
	+12·0	−20·3		+11·8	−20·0
12 37			12 46		
	+12·1	−20·2		+11·9	−19·9
12 55			13 05		
	+12·2	−20·1		+12·0	−19·8
13 14			13 24		
	+12·3	−20·0		+12·1	−19·7
13 35			13 45		
	+12·4	−19·9		+12·2	−19·6
13 56			14 07		
	+12·5	−19·8		+12·3	−19·5
14 18			14 30		
	+12·6	−19·7		+12·4	−19·4
14 42			14 54		
	+12·7	−19·6		+12·5	−19·3
15 06			15 19		
	+12·8	−19·5		+12·6	−19·2
15 32			15 46		
	+12·9	−19·4		+12·7	−19·1
15 59			16 14		
	+13·0	−19·3		+12·8	−19·0
16 28			16 44		
	+13·1	−19·2		+12·9	−18·9
16 59			17 15		
	+13·2	−19·1		+13·0	−18·8
17 32			17 48		
	+13·3	−19·0		+13·1	−18·7
18 06			18 24		
	+13·4	−18·9		+13·2	−18·6
18 42			19 01		
	+13·5	−18·8		+13·3	−18·5
19 21			19 42		
	+13·6	−18·7		+13·4	−18·4
20 03			20 25		
	+13·7	−18·6		+13·5	−18·3
20 48			21 11		
	+13·8	−18·5		+13·6	−18·2
21 35			22 00		
	+13·9	−18·4		+13·7	−18·1
22 26			22 54		
	+14·0	−18·3		+13·8	−18·0
23 22			23 51		
	+14·1	−18·2		+13·9	−17·9
24 21			24 53		
	+14·2	−18·1		+14·0	−17·8
25 26			26 00		
	+14·3	−18·0		+14·1	−17·7
26 36			27 13		
	+14·4	−17·9		+14·2	−17·6
27 52			28 33		
	+14·5	−17·8		+14·3	−17·5
29 15			30 00		
	+14·6	−17·7		+14·4	−17·4
30 46			31 35		
	+14·7	−17·6		+14·5	−17·3
32 26			33 20		
	+14·8	−17·5		+14·6	−17·2
34 17			35 17		
	+14·9	−17·4		+14·7	−17·1
36 20			37 26		
	+15·0	−17·3		+14·8	−17·0
38 36			39 50		
	+15·1	−17·2		+14·9	−16·9
41 08			42 31		
	+15·2	−17·1		+15·0	−16·8
43 59			45 31		
	+15·3	−17·0		+15·1	−16·7
47 10			48 55		
	+15·4	−16·9		+15·2	−16·6
50 46			52 44		
	+15·5	−16·8		+15·3	−16·5
54 49			57 02		
	+15·6	−16·7		+15·4	−16·4
59 23			61 51		
	+15·7	−16·6		+15·5	−16·3
64 30			67 17		
	+15·8	−16·5		+15·6	−16·2
70 12			73 16		
	+15·9	−16·4		+15·7	−16·1
76 26			79 43		
	+16·0	−16·3		+15·8	−16·0
83 05			86 32		
	+16·1	−16·2		+15·9	−15·9
90 00			90 00		

Fig. 2.15 Main correction for apparent altitudes of the sun above the horizon.

2.8 Summary: Constituents of the Main Correction (approximate values)

The corrections in the summary table, below, give the approximate average value of the individual errors, the sign of the required correction (+ or –), and show how they vary with the altitude of the sun above the horizon.

Sun Altitude Ha	Lower Limb to Sun Center	Refraction	Parallax	Main Correction (Total)
10°	+16.0'	–5.1'	+0.15'	+10.8'
20°	+16.0'	–2.5'	+0.05'	+13.5'
80°	+16.0'	–0.1'	+0.00'	+15.9'

Fig. 2.16

The various horizons and sight angles are summarized in the graphic given below in fig. 2.17

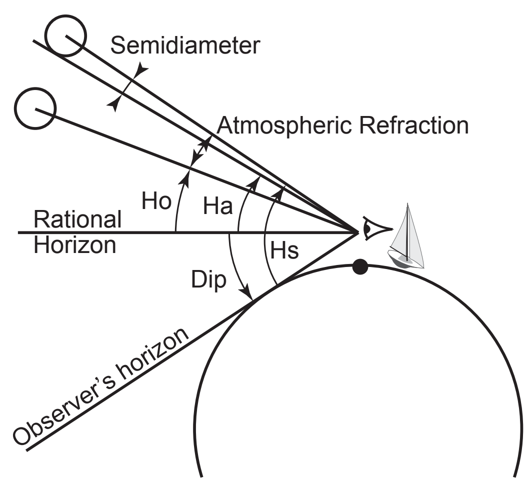


Fig. 2.17 After correction for Index Error, sights taken with the sextant need to be corrected for Dip and atmospheric refraction (shown on graph). In addition, they must be corrected also for the semidiameter of the celestial object, and for parallax in the case of the bodies closest to Earth, such as the Sun, the Moon, Mars and Venus.

OCT.—MAR.	SUN		APR.—SEPT.		
App. Alt.	Lower Limb	Upper Limb	App. Alt.	Lower Limb	Upper Limb
° '	'	'	° '	'	'
9 34			9 39		
	+10·8	−21·5		+10·6	−21·2
9 45			9 51		
	+10·9	−21·4		+10·7	−21·1
9 56			10 03		
	+11·0	−21·3		+10·8	−21·0
10 08			10 15		
	+11·1	−21·2		+10·9	−20·9
10 21			10 27		
	+11·2	−21·1		+11·0	−20·8
10 34			10 40		
	+11·3	−21·0		+11·1	−20·7
10 47			10 54		
	+11·4	−20·9		+11·2	−20·6
11 01			11 08		
	+11·5	−20·8		+11·3	−20·5
11 15			11 23		
	+11·6	−20·7		+11·4	−20·4
11 30			11 38		
	+11·7	−20·6		+11·5	−20·3
11 46			11 54		
	+11·8	−20·5		+11·6	−20·2
12 02			12 10		
	+11·9	−20·4		+11·7	−20·1
12 19			12 28		
	+12·0	−20·3		+11·8	−20·0
12 37			12 46		
	+12·1	−20·2		+11·9	−19·9
12 55			13 05		
	+12·2	−20·1		+12·0	−19·8
13 14			13 24		
	+12·3	−20·0		+12·1	−19·7
13 35			13 45		
	+12·4	−19·9		+12·2	−19·6
13 56			14 07		
	+12·5	−19·8		+12·3	−19·5
14 18			14 30		
	+12·6	−19·7		+12·4	−19·4
14 42					
	+12·7	−19·6		[illegible]	−19·3
[illegible] 06					
	+12·8	[illegible]			[illegible]

Fig. 2.18

2.8.1 Example 1: Winter; Sun low above the horizon, Lower Limb; apparent altitude after correction of Index Error and Dip: Ha 13° 18.2'

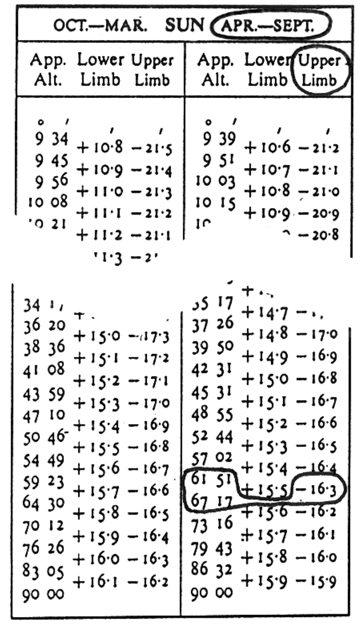
Main correction, from Almanac, Lower Limb, winter months (Oct.–Mar.), for Sun Altitudes between 13° 14' and 13° 35': + 12.3' (fig. 2.18)

Sextant altitude corrected for semidiameter, refraction, and parallax (Main Correction):

Ha	13° 18.2'
Main correction	+ 12.3'
Ho	13° 30.5'

2.8.2 Example 2: Summer; Sun high above the horizon, Upper Limb; apparent altitude: Ha 62° 12.5'

Main Correction, from Almanac, Upper Limb, summer months (Apr.–Sept.), for Sun Altitudes between 61° 51' and 67°17': – 16.3' (fig. 2.19)



OCT.—MAR.	SUN		APR.—SEPT.		
App. Alt.	Lower Limb	Upper Limb	App. Alt.	Lower Limb	Upper Limb
° '	'	'	° '	'	'
9 34			9 39		
	+10·8	−21·5		+10·6	−21·2
9 45			9 51		
	+10·9	−21·4		+10·7	−21·1
9 56			10 03		
	+11·0	−21·3		+10·8	−21·0
10 08			10 15		
	+11·1	−21·2		+10·9	−20·9
[illegible] 21			[illegible]		
	+11·2	−21·1		[illegible]	−20·8
	[illegible]1·3	−2[illegible]			
34 [illegible]			[illegible]5 17		
	+ [illegible]	[illegible]		+14·7	− [illegible]
36 20			37 26		
	+15·0	−17·3		+14·8	−17·0
38 36			39 50		
	+15·1	−17·2		+14·9	−16·9
41 08			42 31		
	+15·2	−17·1		+15·0	−16·8
43 59			45 31		
	+15·3	−17·0		+15·1	−16·7
47 10			48 55		
	+15·4	−16·9		+15·2	−16·6
50 46			52 44		
	+15·5	−16·8		+15·3	−16·5
54 49			57 02		
	+15·6	−16·7		+15·4	−16·4
59 23			61 51		
	+15·7	−16·6		+15·5	−16·3
64 30			67 17		
	+15·8	−16·5		+15·6	−16·2
70 12			73 16		
	+15·9	−16·4		+15·7	−16·1
76 26			79 43		
	+16·0	−16·3		+15·8	−16·0
83 05			86 32		
	+16·1	−16·2		+15·9	−15·9
90 00			90 00		

Fig. 2.19

Sextant altitude corrected for semidiameter, refraction, and parallax (Main Correction):

Ha	62° 12.5' =	61° 72.5'
Main correction		– 16.3'
Ho		61° 56.2'

2.9 Examples showing all corrections: Index Error, Dip, and Main Correction

For these examples, use the *Altitude Correction Tables*, Appendix 2, p. 1.

2.9.1 Example 1: Oct.–Mar.; Sun low above the horizon, Index Error off the arc; Height of eye in feet; Sun Lower Limb

Month:	January
Sextant altitude Hs:	19° 01.2'
Index Error (off the arc):	2.4'
Height of the eye:	18 ft

Answers:

Hs	19° 01.2'		
Index Error	+ 2.4'		
Hs corrected for index error	19° 03.6'	=	18° 63.6'
Dip (18 ft)			– 4.1'
Apparent Altitude Ha			18° 59.5'
Main Correction (Oct.–Mar.)			+ 13.5'
Observed Altitude Ho			19° 13.0'

2.9.2 Example 2: Summer; Sun high above the horizon, Index Error on the arc; Height of eye in metres; Sun Upper Limb

Month:	July
Sextant altitude Hs:	47° 17.5'
Index Error (on the arc):	3.2'
Height of the eye:	3.0 m

Answers:

Hs	47° 17.5'		
Index Error	– 3.2'		
Hs corrected for index error	47° 14.3'		
Dip (3.0 m)	– 3.0'		
Apparent Altitude Ha	47° 11.3'	=	46° 71.3'
Main Correction (Apr.–Sept.)			– 16.7'
Observed Altitude Ho			46° 54.6'

2.10 Exercises: Sextant Corrections

For these exercises, use the full *Altitude Correction Tables* of the Almanac, Appendix 2, p. 1)

Summer and winter months; Upper and Lower Limb. Calculate the Observed Altitude Ho.

Body observed	Sun, LL	Sun, UL	Sun, LL	Sun, UL
Time of year	May	June	December	January
Sextant Altitude Hs	47° 41.0'	52° 01.0'	18° 56.0'	22° 16.0'
Index Error	3' Off	2' On	5' Off	6' On
Height of eye	2.9 m	3.3 m	9 feet	12 feet

Hs corrected for Index Error On – Off +				
Dip	–	–	–	–
Apparent Altitude Ha				
Main Correction LL + UL –				
Observed Altitude Ho				

Answers:

Hs corrected for Index Error On – Off +	47° 44.0'	51° 59.0'	19° 01.0'	22° 10.0'
Dip	– 3.0'	– 3.2'	– 2.9'	– 3.4'
Apparent Altitude Ha	**47° 41.0'**	**51° 55.8'**	**18° 58.1'**	**22° 06.6'**
Main Correction LL + UL –	+ 15.1'	– 16.6'	+ 13.5'	– 18.4'
Observed Altitude Ho	**47° 56.1'**	**51° 39.2'**	**19° 11.6'**	**21° 48.2'**

2.11 Adjustment of the Sextant Mirrors

The altitude of the celestial bodies above the horizon, as measured with a sextant, is meaningful only if the mirrors have been adjusted properly and the Index Error recorded precisely. This section explains how this is done.

2.11.1 Adjustment of the Index Mirror

The Index Mirror is easy to adjust. It needs to be exactly perpendicular to the frame of the sextant, i.e. the plane of the Index Arm and the Arc. This adjustment is done first, using the single screw on the side of the Index Mirror.

The arm of the sextant is rotated so that the index is somewhere along the first quarter of the arc, between 0º and 30º (further out on certain sextants, perhaps to 60º). If we then place our eye near the plane of the sextant, close to the Index Mirror and its filters (pushed down and out of the way), we can see two sectors of the Arc (fig. 2.20).

The first visible sector is the one which is reflected in the Index Mirror; it is an image of the end of the arc, with graduations of perhaps 100º to 120º. The second sector, just to the right of the Index Mirror, is visible directly; we should see the beginning of the arc, and the graduations before and/or after the zero on the arc.

We turn the single screw on the Index Mirror so that the two sectors of the arc, i.e. the one we see in the mirror and the one directly to the right of the mirror, are contiguous, in line with each other. The Index Mirror is then perpendicular to the frame of the sextant (fig. 2.21).

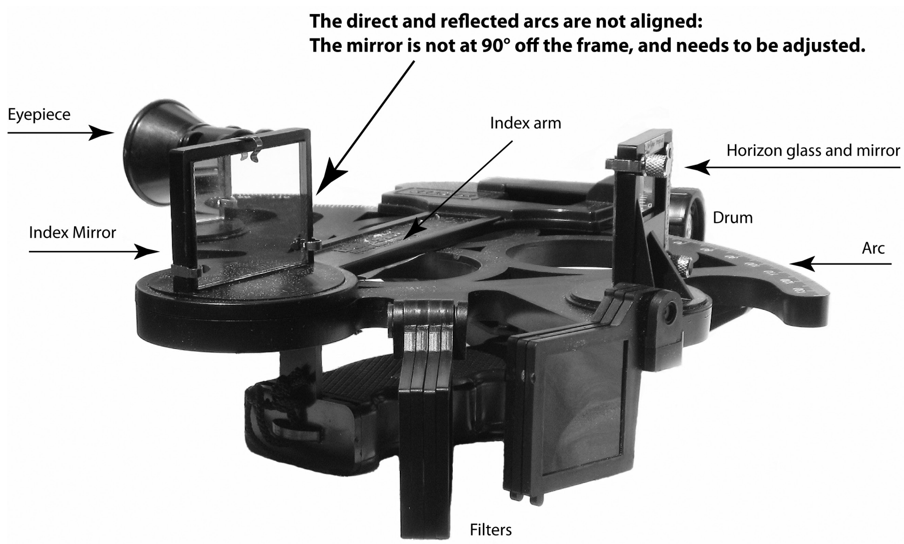


Fig. 2.20 Index Mirror NOT perpendicular to the sextant frame.

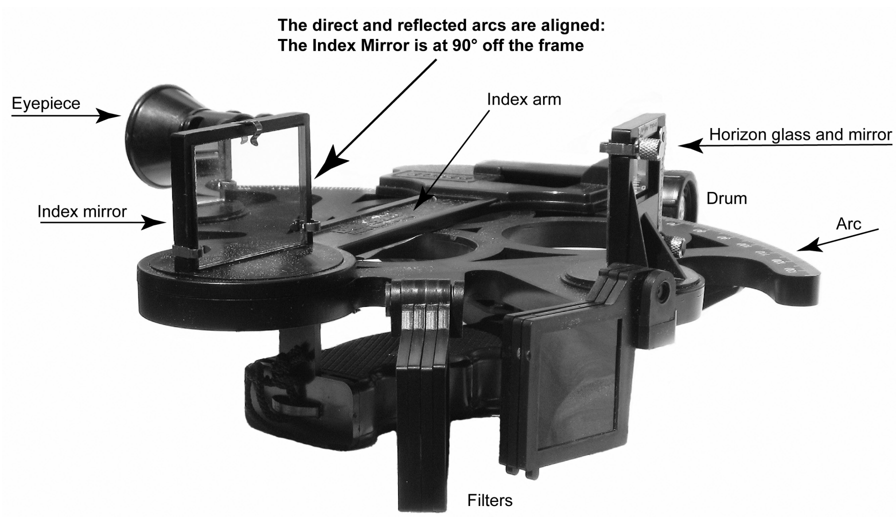


Fig. 2.21 Index Mirror perpendicular to the sextant frame.

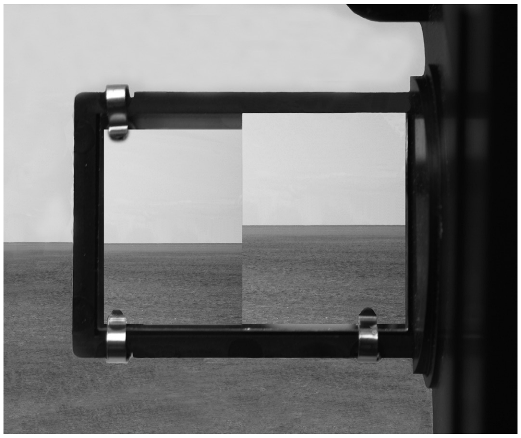


Fig. 2.22 The Horizon Mirror needs to be adjusted for altitude error with the top screw.

Fig. 2.23 Horizon Mirror adjusted for altitude error.

2.11.2 Adjustment of the Horizon Mirror

The Horizon Mirror is a little more difficult to adjust. When properly set, the Horizon Mirror reflects the image of the celestial body (reflected first in the Index Mirror) exactly straight in front of the eye and towards the horizon, neither Up nor Down (altitude error), or Left or Right (side error). We need the two screws of the Horizon Mirror for this double adjustment. Most sextants are designed to be held in the right hand.

First, we set the index to exactly zero, with the arm at 0º, and the drum (see fig. 2.2, p. 11) at 0'. The most important adjustment is Up and Down, which sets the reference for altitude. Looking through the eyepiece (fig. 2.22), we see two horizons: one directly, just left of the Horizon Mirror, through the clear window or open space; and one in the Horizon Mirror itself. The direct and reflected horizons should be exactly at the same level (fig. 2.23). We make any Up or Down adjustment required by tilting the Horizon Mirror forward or backward with its top screw.

Once the real and reflected horizons are side by side, we need to correct any side error (fig. 2.24). There are usually no vertical surfaces at sea, so that only an approximate adjustment can be made by tilting the sextant 45º and using the horizon again. Any discontinuity of the direct and reflected horizons can be corrected with the side screw (fig. 2.25).

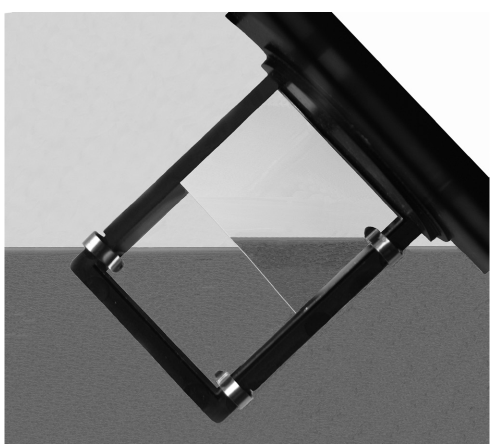


Fig. 2.24 The Horizon Mirror needs to be adjusted for side error with the side screw.

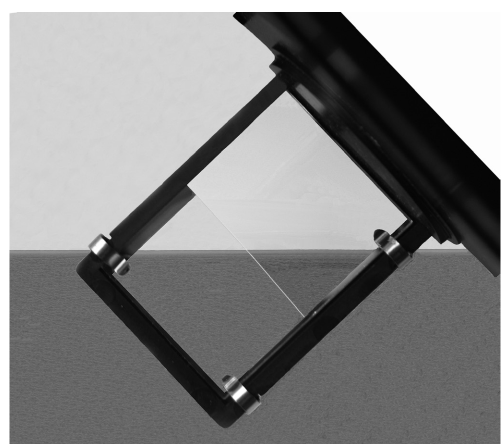


Fig. 2.25 Horizon Mirror adjusted for side error.

Correcting the side error re-introduces a small altitude error. The sextant should then be returned to its normal (vertical) position, and any altitude error caused by the previous adjustment should be neutralized with the top screw. When the altitude error is not fully corrected by an adjustment of the mirrors, the remaining error must be recorded: it is the Index Error.

Another (clever) method for adjusting the horizon mirror was suggested by one of my students: with the index at 0° and all the filters in place, look at the sun through the eyepiece. Since the horizon mirror is not adjusted, two suns are visible, close to each other. Adjust the two screws on the horizon mirror to superimpose the reflected sun onto the direct sun. This can be a little tricky because the filters are in the way, and the mirrors might be a little wobbly, especially on sextants whose frame is not perfectly rigid. When the two suns are one, the mirrors are exactly parallel to each other. This overlay method can also be used at night with the stars or the planets.

2.12 Taking a Sight on the Sun with a Sextant

A sight on the sun, or any other celestial body, is simply the measure of its altitude, i.e. its height (angle) above the horizon. Once its mirrors are adjusted, a sextant is the perfect instrument to measure this angle.

Experience proves that the easiest method to take a sight is as follows:

➢ Pushing the index far forward, e.g. around 60° or more, to avoid seeing the sun in the mirrors, place a couple of filters (for instance two dark filters initially) in front of the horizon mirror and the clear opening next to it, and take a quick glance at the sun through both the eyepiece and the clear opening. It is safest to use too many filters initially. Adjust the filters if required, down to possibly a blue and an orange. Be extremely careful not to allow the sun directly into your eye: severe damage to your eyesight could occur.

➢ Place the same types of filters in front of the index mirror. The eye is now protected from both the direct sun light coming through the clear opening next to the horizon mirror, and from the image of the sun reflected in the index and horizon mirrors.

➢ Rotate the drum to show 0', and the index to show 0°. The two mirrors should be parallel, and the observer should see, in the eyepiece, two images, very close to each other, of a distant object: one is the direct image seen through the opening next to the horizon mirror; the other is its reflected image. For instance, two suns, either green, orange or blue depending on the filters used, should thus be visible next to each other; they might even be superimposed, if the mirrors are exactly parallel.

➢ The delicate part starts here: ignoring the direct sun, we bring the reflected sun down towards the horizon. This requires pushing the index forward very slowly and gently, while rotating the sextant down towards the horizon in order to keep the reflected sun visible in the mirrors.

➢ As the sextant is slowly rotated down, the horizon will eventually appear in the opening next to the horizon mirror. The sun is still visible in the mirrors. The trick consists in placing the image of the sun right on the horizon. The altitude of the sun is simply the angle read on the index and drum.

A few practical points should be noted:

- When the sextant is rotated to about halfway down towards the horizon, it is usually necessary to remove one or both filters from the horizon mirror. The horizon is usually not very bright, unless the sun is reflected on the water, and could disappear behind the filters.
- If the horizon mirror is semi-transparent, the sun can be brought easily right onto it. If, as is the case for most sextants, the horizon mirror is a traditional one, then the horizon is seen to the left, through the opening, while the sun is seen to the right, in the mirror. The sun, therefore, cannot be placed directly **on** the horizon, but will always be next to it, a little to the right. In order to insure that the sun is exactly level with the horizon, the observer needs to swing the sextant left and right: the sun oscillates along the horizon like a pendulum, and the index angle can then be fine-tuned with the drum so that the bottom of this pendulum trajectory appears to be tangent to the horizon.
- In the case of a sight on the sun, the Lower Limb is traditionally used, and is the one placed on the horizon. Occasionally, clouds get in the way, and the Upper Limb must be used. In the case of the moon, the limb used depends on the phase. The stars and planets are considered punctual.