Chapter 6: Latitude by Noon Sight

When the sun is crossing the meridian of the boat, it is straight south or north of the boat and at its highest altitude over the horizon for the day. The local meridian time is 12:00 (plus or minus up to 16 min, depending on the time of year).

At that moment, assuming we know the UTC time precisely, we read in the Almanac the coordinates (Dec and GHA) of the sun's GP. The sun's GHA at noon (local meridian time) coincides with the longitude of our boat. Its Declination is the latitude of the sun's GP.

Knowing the sun's Declination, i.e. the latitude (north or south of the equator) of the sun's GP, and measuring with the sextant how far north or south of the sun's GP we are, we can calculate our own latitude. For instance, if the sun's GP is north of the equator (summer months) and we are north of the sun's GP (sailing, for instance, in the North Pacific), our latitude is simply the Declination of the sun plus our distance north of the sun's GP (fig. 6.1). These calculations are discussed in the following sections.

6.1 Calculation of Latitude, given the Sun Altitude and Declination

During a noon sight, the boat and the sun are exactly on the same meridian: the sun is either exactly north or exactly south of the boat. At a time of the year when the sun is over the equator (Equinox), the latitude of a boat is simply the **zenith distance ZD** of the sun (the angle between the sun and the vertical, or **90°– Ho**; fig. 6.1). The Circle of Position is centered on the Geographic Position (**GP**) of the sun (fig. 6.2).

In the general case, the sun is not over the equator. The latitude of the center of the Circle of Position, i.e. the latitude of the sun's GP, is simply the Declination of the sun, north or south depending on the time of year (fig. 6.3).

The latitude of the boat at noon is then:

Noon Lat. = ZD +/– Sun Dec

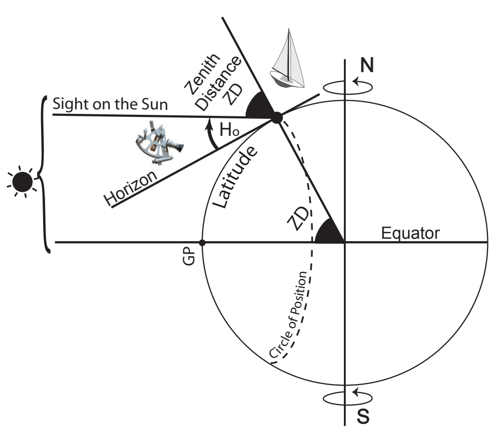


Fig. 6.1 Sun over the equator (Equinox): Lat. = ZD, the Zenith Distance (ZD = 90° – Ho).

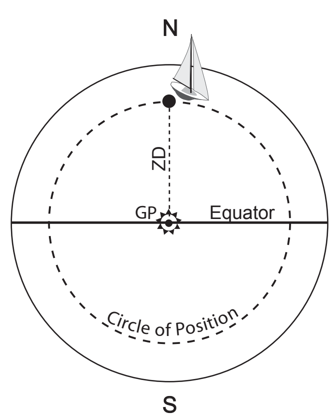


Fig. 6.2 Circle of Position from a noon sight at a time of year when the Sun is over the equator (Equinox).

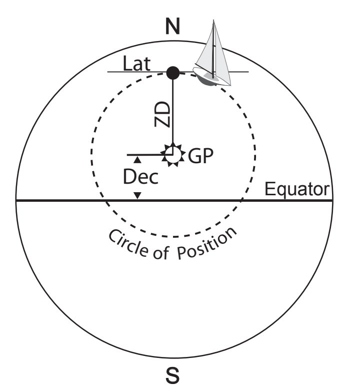


Fig. 6.3 Circle of Position from a noon sight at a time of year when the sun is not over the equator (general case): The latitude of the center of the Circle of Position (sun's GP) is the latitude of the sun's GP, i.e. the sun's Declination.

6.1.1 Scenario 1: Sun GP and boat in same hemisphere; boat outside of the Tropics

When the boat and the sun GP are both on the same side of the equator (Boat Latitude and sun Declination are of the **same name**), and when the boat is further away from the equator than the sun GP (Lat. > Dec), the 1atitude of the boat, from a noon sight, is equal to the Zenith Distance + the sun Declination (fig. 6.1).

In the *Sight Reduction Tables*, is a scenario called: *Dec and Lat. "same name"; Lat. bigger than Dec*:

Lat. (Noon Sight) = (90° – Ho) + Dec = **ZD + Dec**

Example (Declination "same name" as Latitude)

The boat is in the northern hemisphere and north of the Tropic of Cancer, during the summer: the GP of the sun is also north of the equator.

Lat. = ZD + Dec	
Date and time of noon sight:	July 30, 2003 at 21:24 UTC
Sextant altitude Ho at noon:	53° 28.2'

a) Sun Dec (from Almanac, Appendix 2 p. A2-10)

Sun Declination @ 21:00	18° 27.1' N
Change in Dec for 1h: d = –0.6'	
Interpolation for 24 min*	– 0.2'
Dec of the sun at 21:24	18° 26.9' N

*Estimated as a little over 1/3 of the 0.6 min per hour; or read off the Almanac page for 24 min, Appendix 2 p. A2-20, *v* or *d* column for d = 0.6.

b) Zenith Distance ZD = 90° – Ho

90°	89° 60.0'
– Ho	– 53° 28.2'
ZD	36° 31.8'

c) Lat. = ZD + Dec

ZD	36° 31.8'
+ Dec of sun	+ 18° 26.9'
Latitude	54° 58.7' N

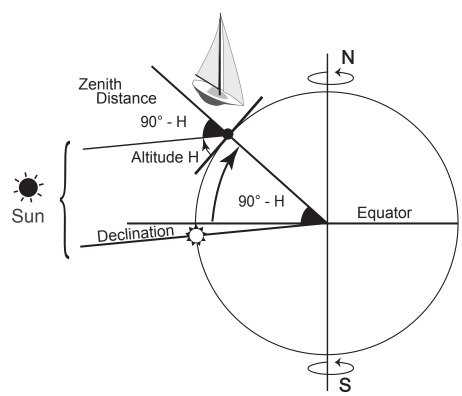


Fig. 6.4 Sun GP and Boat in opposite hemispheres: Lat. = ZD – Dec.

6.1.2 Scenario 2: Sun GP and boat in opposite hemispheres

This is an example of latitude and Declination with **contrary names** (fig. 6.2). When the boat is on one side of the equator, and the sun on the other, the latitude of the boat, during a noon sight, is equal to ZD – the Declination of the sun. In this case, called: *Dec and Lat. contrary names* in the *Sight Reduction Tables*,

Lat. (Noon Sight) = ZD – Dec

6.1.3 Scenario 3: Sailing in the Tropics, the boat is between the sun and the equator

During a sail in the tropics, the sun might be to the north of the boat, and the equator to the south, or the other way around (fig. 6.5). During a noon sight, the latitude of the boat is equal to the Declination of the sun – the Zenith Distance of the sun. In this case, called *Dec and Lat. "Same names"; Lat. smaller than Dec* in the *Sight Reduction Tables*:

Lat. (Noon Sight) = Dec – ZD

The formulae for calculating the latitude from a noon sight are reproduced at the bottom of the work forms given in Appendix 1

Lat. and Dec same name, and Lat. > Dec: Lat. = ZD + Dec;
Lat. and Dec same name, and Lat. < Dec: Lat. = Dec – ZD;
Lat. and Dec contrary names: Lat. = ZD – Dec

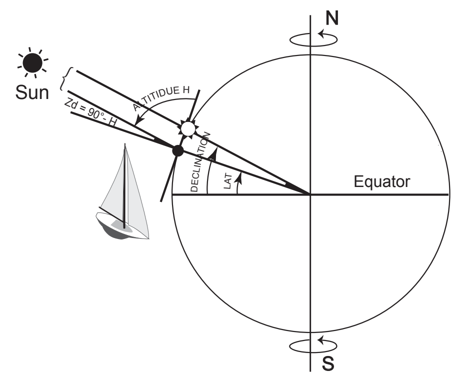


Fig. 6.5 Boat in the tropics; sun further away from the equator than the boat: Lat. = Dec – ZD.

6.2 Plotting the Sun Trajectory in order to measure the Sun Altitude at Noon

The top of the trajectory of the sun though the sky around noon is fairly flat. For the observer, therefore, it is quite difficult to tell whether the sun has reached its maximum altitude or not. A more precise estimate of the sun's maximum altitude can be obtained from a series of sights with the sextant shortly before and after noon (fig. 6.6). From the maximum altitude of the sun, we can calculate our latitude, and from the (not very precise) time when the maximum altitude was reached, we can tell our approximate longitude (see Chapter 7).

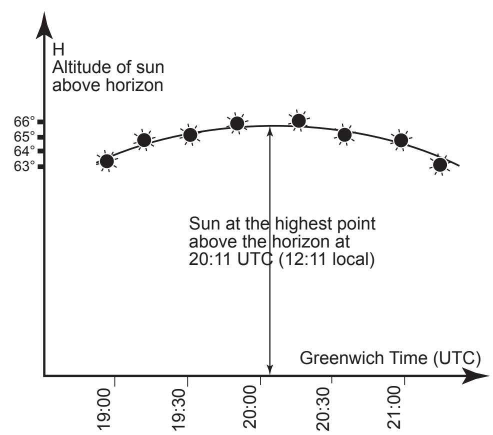


Fig. 6.6 A series of sun shots in the late morning, around noon, and in the early afternoon provide a reasonably accurate measure of the maximum altitude of the sun.

6.3 Exercises

For the following sights, determine the sun Declination; The formula (+/– ZD +/– Dec) for the calculation of the boat latitude; the sun's Zenith Distance; the latitude of the boat; its approximate longitude; and the part of the world where the boat is sailing.

Date of the noon sight (UTC)	April 21	August 22	Nov. 20
Time of the noon sight (UTC)	21:49	20:21	08:48
Approximate latitude of the boat	15° S	45° N	10° S
Sextant altitude Ho of the sun at noon	62° 12.3'	57° 50.7'	79° 31.2'

Sun Declination

Sun Dec for the hour of sight			
Change in sun Dec per hour			
Interpol. of Dec for the min of sight			
Sun Dec for exact the time of sight			

Zenith Distance

90°	89° 60.0'	89° 60.0'	89° 60.0'
– Ho			
ZD			

Formula

Dec and Lat. **Same** or **Contrary** names			
If **same** names, Lat. “ **>** ” or “ **<** ” than Dec			
Formula for Lat. (+/– ZD +/– Dec)			

Latitude of boat

ZD or Dec			
+/– Dec or ZD			
Boat latitude			

Approximate longitude of boat (@ 15°/h, assuming the sun crossed the Greenwich Meridian at exactly 12:00 UTC), and area of sailing

Approximate longitude			
Area of Sailing			

Answers:

Sun Declination

Sun Dec for the hour of sight	11° 56.4' N	11° 42.8' N	19° 37.1' S
Change in sun Dec per hour	(+0.9' per 60 min)	(–0.8' per 60 min)	(+0.6' per 60 min)
Interpolation of Dec for the min of sight	+ 0.7'	–0.3'	+ 0.5'
Sun Dec for exact the time of sight	**11° 57.1' N**	**11° 42.5' N**	**19° 37.6' S**

Zenith Distance

90°	89° 60.0'	89° 60.0'	89° 60.0'
– Ho	– 62° 12.3'	– 57° 50.7'	– 79° 31.2'
ZD	**27° 47.7'**	**32° 09.3'**	**10° 28.8'**

Formula

Dec and Lat. **Same** or **Contrary** names	Contrary	Same	Same
If **same** names, Lat. “ **>** ” or “ **<** ” than Dec		>	<
Formula for Lat. (+/– ZD +/– Dec)	ZD – Dec	ZD + Dec	Dec – ZD

Latitude of boat

ZD or Dec	27° 47.7'	32° 09.3'	19° 37.6'
+/– Dec or ZD	– 11° 57.1'	+ 11° 42.5'	– 10° 28.8'
Boat latitude	**15° 50.6' S**	**43° 51.8' N**	**09° 08.8' S**

Approximate longitude of boat (@ 15°/h, assuming the sun crossed the Greenwich Meridian at exactly 12:00 UTC), and area of sailing

Approximate longitude	147° W	125° W	048° E
Area of Sailing	Tahiti	off Oregon Coast	Seychelles

6.4 Review Exercise: Traditional plotting method, before Marcq Saint Hilaire

The traditional Celestial Navigation method for plotting a boat location on a world chart is to draw circles of position centered on the Geographic Positions of the sun a few hours apart, with a radius equal to **90° – Ho**. The GPs of the sun (or any celestial object) are determined from the Nautical Almanac, and Ho is, for each sight, the height of the sun, measured with the sextant (Sections 1.4 and 1.7).

The purpose of this exercise is to illustrate the method and show its shortcomings. As mentioned at the end of Section 1.4, the main drawback of this approach is the lack of precision of the boat position determined from the intersection of circles which, typically, have radii of several thousand of nautical miles. The Marcq Saint Hilaire method, described in the following sections together with the associated *Sight Reduction Tables*, alleviates the problem by comparing the sun's altitude above the horizon, as measured with the sextant, and the altitude calculated from the *Sight Reduction Tables*. This Calculated Altitude Hc tells the navigator at what height the sun would be above the horizon if the boat were exactly at its **assumed position**. The difference between the measured and calculated altitudes, in minutes of angle, is simply the distance, in nautical miles, between the assumed position and the real line of position.

The exercise relies on the traditional morning, noon, and afternoon sights, and on the type of plotting used until the end of the 19th century when the Marcq Saint Hilaire method became widely used. There are no sights on Polaris since, in the Southern Hemisphere, the star is not visible and cannot be used to determine latitude.

Exercise: While exploring the South Pacific in 1769, Captain Cook took the following sights on the sun:

1. At 19:00 UTC, June 21: Ho = 30°
2. At 22:00 UTC, June 21: Ho = 48°
3. At 01:00 UTC, June 22: Ho = 30°

Without using any table, and rounding off angles and times to the nearest degree or hour, plot the lines and circles of position on the world chart given next page, and determine Captain Cook's position.

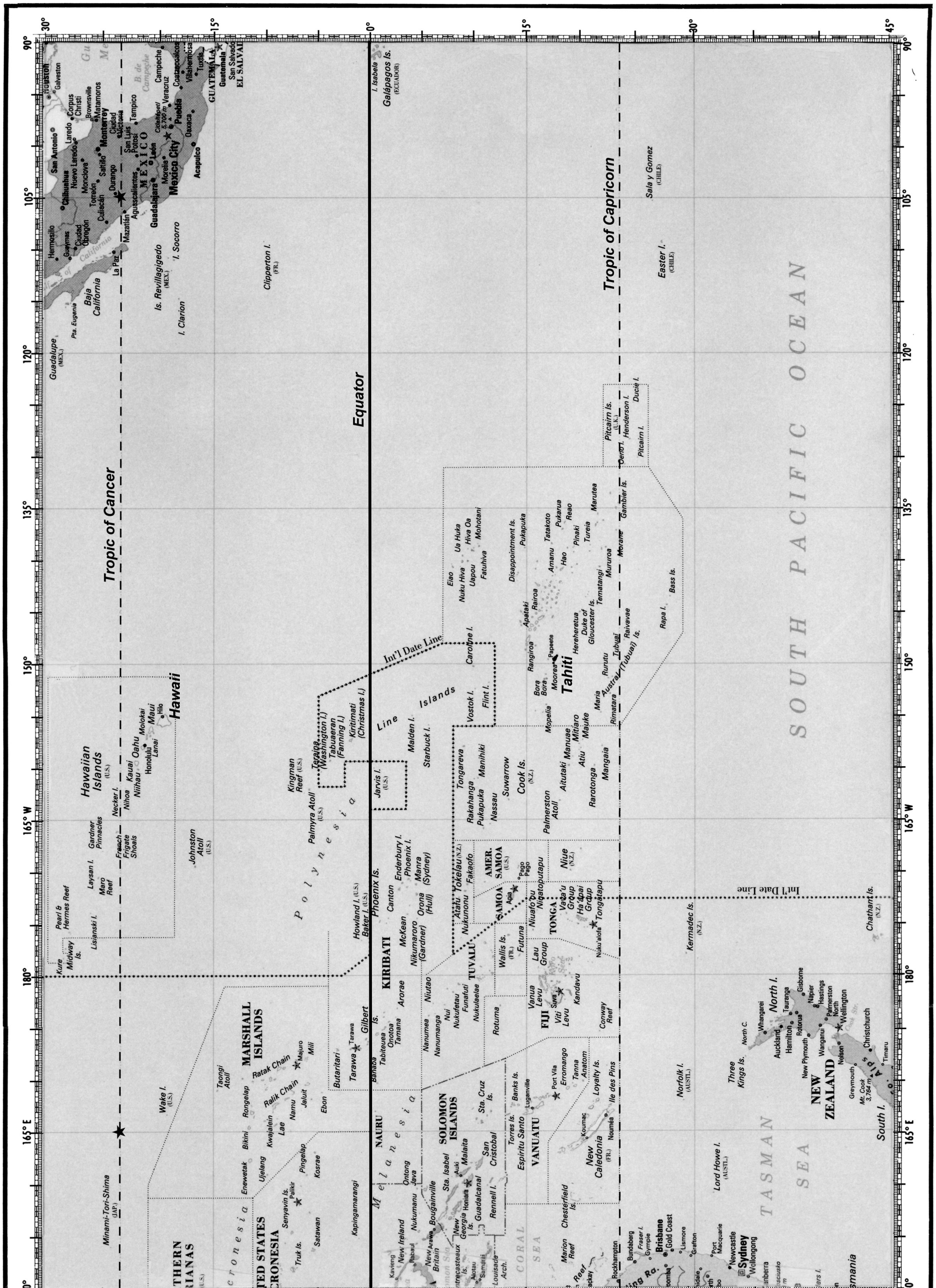
Answer: On June 21 and 22, during the summer solstice, the sun Declination is at its maximum, a little over 23° N (Section 4.1). At 19:00 UTC, the sun's GP is therefore at Lat. 23° N. Assuming that the sun crossed the Greenwich Meridian at 12:00 UTC on that day, we can see that it has travelled (19:00 – 12:00) x 15°/h = 105° W by the time we take the **morning sight** at 19:00 UTC. The longitude of the sun's GP is therefore 105° W.

During the **afternoon sight**, the sun's GP is again at Lat. 23° N. In order to calculate its longitude, we note that, at 01:00 UTC, the sun has 11 hours to go before reaching the meridian of Greenwich at 12:00. The sun is therefore over the meridian at longitude 11 x 15°/h = 165° E.

The **noon sight** is taken when the sun is highest above the southern horizon, at 22:00 UTC. At that time, the sun crosses the boat meridian. It has travelled 22:00 – 12:00 = 10 hours since it crossed the Greenwich Meridian, and it is therefore over a meridian at 10:00 x 15°/h = 150° longitude W. This is the longitude of the boat.

The latitude of the boat from the noon sight is the Zenith Distance adjusted for the Declination of the sun, i.e. **ZD – Dec.** (Section 6.1.2) = (90°– 48°) – 23° = 19° S.

If we trace two circles of position centered on the parallel of 23° N, and on the longitudes of 105° W and 165° E, with radii of (90°– 30°) x 60'/° = 3,600' or 3,600 NM, we note that they intersect just south of Tahiti. The latitude is 19° S, and the longitude 150° W (fig. 6.7).



Tropic of Cancer
Equator
Tropic of Capricorn
SOUTH PACIFIC OCEAN
Hawaii
Hawaiian Islands
Tahiti
MEXICO
KIRIBATI
MARSHALL ISLANDS
SOLOMON ISLANDS
NEW ZEALAND
TASMAN SEA
Int'l Date Line

fig. 6.7